#  REGIONALS 

November 3, 2018

## Contest Problems

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This contest contains ten problems over 24 pages. Good luck.

For problems that state "Your answer should have an absolute error of less than $10^{-9}$ ", your answer, $x$, will be compared to the correct answer, $y$. If $|x-y|<10^{-9}$, then your answer will be considered correct.

## Definition 1

For problems that ask for a result modulo $m$ :
If the correct answer to the problem is the integer $b$, then you should display the unique value $a$ such that:

- $0 \leq a<m$
and
- $(a-b)$ is a multiple of $m$.


## Definition 2

A string $s_{1} s_{2} \cdots s_{n}$ is lexicographically smaller than $t_{1} t_{2} \cdots t_{\ell}$ if

- there exists $k \leq \min (n, \ell)$ such that $s_{i}=t_{i}$ for all $1 \leq i<k$ and $s_{k}<t_{k}$
or
- $s_{i}=t_{i}$ for all $1 \leq i \leq \min (n, \ell)$ and $n<\ell$.


## Definition 3

- Uppercase letters are the uppercase English letters $(A, B, \ldots, Z)$.
- Lowercase letters are the lowercase English letters $(a, b, \ldots, z)$.
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# Problem A Quality-Adjusted Life-Year <br> Time limit: 1 second 

The Quality-Adjusted Life-Year (QALY) is a way to measure a person's quality of life that includes both the quality and the quantity of life lived.

The quality of life lived can be quantified as a number between 0 and 1. If someone is living with perfect health, the quality of life is 1 . If
 someone is dead, then the quality of life is 0 . The quality of life may increase or decrease due to medical treatements, sickness, etc.

The QALY for each period in which the quality of life is constant is simply the product of the quality of life and the length of the period (in years). We wish to know the amount of QALY accumulated by a person at the time of death, given the complete history of this person.

## Input

The first line of input contains a single integer $N(1 \leq N \leq 100)$, which is the number of periods of constant quality of life during the person's lifetime.

The next $N$ lines describe the periods of life. Each of these lines contains two real numbers $q(0<q \leq 1)$, which is the quality of life in this period, and $y(0<y \leq 100)$, which is the number of years in this period. All real numbers will be specified to exactly one decimal place.

## Output

Display the QALY accumulated by the person. Your answer will be considered correct if its absolute error does not exceed $10^{-3}$.

## Sample Input 1 <br> Sample Output 1

| 5 |  | 41.470 |
| :--- | :--- | :--- |
| 1.0 | 12.0 |  |
| 0.7 | 5.2 |  |
| 0.9 | 10.7 |  |
| 0.5 | 20.4 |  |
| 0.2 | 30.0 |  |

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# Problem B <br> Gwen's Gift <br> Time limit: 1 second 

Gwen loves most numbers. In fact, she loves every number that is not a multiple of $n$ (she really hates the number $n$ ). For her friends' birthdays this year, Gwen has decided to draw each of them a sequence of $n-1$ flowers. Each of the flowers will contain between 1 and $n-1$ flower petals (inclusive). Because of her hatred of multiples of $n$, the total number of petals in any non-empty contiguous subsequence of flowers cannot be a multiple of $n$. For example, if $n=5$, then the top two paintings are valid, while the bottom painting is not valid since the second, third and fourth flowers have a total of 10 petals. (The top two images are Sample Input 3 and 4.)


Gwen wants her paintings to be unique, so no two paintings will have the same sequence of flowers. To keep track of this, Gwen recorded each painting as a sequence of $n-1$ numbers specifying the number of petals in each flower from left to right. She has written down all valid sequences of length $n-1$ in lexicographical order. A sequence $a_{1}, a_{2}, \ldots, a_{n-1}$ is lexicographically smaller than $b_{1}, b_{2}, \ldots, b_{n-1}$ if there exists an index $k$ such that $a_{i}=b_{i}$ for $i<k$ and $a_{k}<b_{k}$.

What is the $k$ th sequence on Gwen's list?

## Input

The input consists of a single line containing two integers $n$ ( $2 \leq n \leq 1000$ ), which is Gwen's hated number, and $k$ ( $1 \leq k \leq 10^{18}$ ), which is the index of the valid sequence in question if all valid sequences were ordered lexicographically. It is guaranteed that there exist at least $k$ valid sequences for this value of $n$.

## Output

Display the $k$ th sequence on Gwen's list.
Sample Input $1 \quad$ Sample Output 1

| 43 | 2 | 1 | 2 |
| :--- | :--- | :--- | :--- |



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## Problem C Forest for the Trees

## Time limit: 1 second

You are playing hide-and-go-seek in a forest with Belle. The forest has one tree at each of the positive integer lattice points. That is, there is a tree at every point $(x, y)$ where $x$ and $y$ are both positive integers. You may consider each tree as a point. A logging company has cut down all of the trees in some axis-aligned rectangle, including those on the boundary of the rectangle.

You are standing at $(0,0)$ and Belle is standing at $\left(x_{b}, y_{b}\right)$. You can see Belle if and only if there is no tree blocking your line of sight to Belle. If there is a tree at $\left(x_{b}, y_{b}\right)$, Belle will make it easier for you to find her by standing on the side of the tree facing your location.

For example, suppose that Belle is standing at $(2,6)$. If the trees in the rectangle with corners at $(1,1)$ and $(5,4)$ are cut down (blue rectangle in figure), then you can see Belle. However, if the rectangle was at $(3,5)$ and $(5,7)$ (red rectangle in figure), then the tree at $(1,3)$ would be in the way.


Given the rectangle and Belle's location, can you see her?

## Input

The first line of input contains two integer $x_{b}$ and $y_{b}\left(1 \leq x_{b}, y_{b} \leq 10^{12}\right)$, which are the coordinates that Belle is standing on.

The second line of input contains four integers $x_{1}, y_{1}, x_{2}$ and $y_{2}\left(1 \leq x_{1} \leq x_{2} \leq 10^{12}\right.$ and $1 \leq y_{1} \leq y_{2} \leq$ $10^{12}$ ), which specify two opposite corners of the rectangle at $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$.

## Output

If you can see Belle, display Yes.
Otherwise, display No and the coordinates of the closest tree that is blocking your view.
Sample Input $1 \quad$ Sample Output 1

| 2 | 6 |  | Yes |
| :--- | :--- | :--- | :--- |
| 1 | 1 | 5 | 4 |

Sample Input 2
Sample Output 2

| 2 | 6 |  |  |
| :--- | :--- | :--- | :--- |
| 3 | 5 | 5 | 7 |No

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\left.| Sample Input 3 | Sample Output 3 |  |  |
| :--- | :--- | :--- | :--- |
| 830844890448 | 39710592053 |  |  |
| 821266 | 42860 | 402207107926 | 423171345006 |$\right) 402207964848 \quad 19223704203$

## Problem D

## H-Index

Time limit: 1 second

In research, it is tough to determine how good of a researcher you are. One way that people determine how good you are is by looking at your H-Index.

Each paper has a certain number of citations. Your $H$-Index is the largest number $H$ such that you have $H$ papers with at least $H$ citations. Given the number of citations on each paper you have written, what is your $H$-Index?


## Input

The first line of input contains a single integer $n(1 \leq n \leq 100000)$, which is the number of papers you have written.

The next $n$ lines describe the papers. Each of these lines contains a single integer $c(0 \leq c \leq 1000000000)$, which is the number of citations that this paper has.

## Output

Display your $H$-Index.

## Sample Input 1 <br> Sample Output 1

| 5 | 2 |
| :--- | :--- |
| 7 |  |
| 1 |  |
| 2 |  |
| 1 |  |
| 5 |  |

Sample Input 2

## Sample Output 2

| 5 | 3 |
| :--- | :--- |
| 7 |  |
| 1 |  |
| 3 |  |
| 1 |  |



## Problem E

Driving Lanes
Time limit: 2 seconds

While driving around a curve on the highway, Sam realizes that if they use the inside lane, they travel a shorter distance. Sam wonders what is the minimum distance needed to travel to the destination.

The multilane highway consists of a sequence of straightaways that are connected by curves. When going around a curve, the distance travelled depends on which lane you are in. Each curve has a curvature $c$ and stretch $s$. Specifically, if Sam is in lane $i$, then they travel $s+c \cdot i$ meters while going around this curve.

Whenever Sam is on a straightaway, they may change from one lane into an adjacent lane. When changing to an adjacent lane, Sam moves forward $k$ meters, but travels a total of $k+r$ meters. Each lane change must be completed before the car reaches the end of the current straightaway. Sam may change lanes multiple times in the same straightaway. For safety reasons, changing lanes is not possible on curves.


Sam starts in lane 1 and wishes to end in lane 1 . What is the minimum distance they must travel?

## Input

The first line of input contains two integers $n(1 \leq n \leq 250)$, which is the number of straightaways, and $m(1 \leq m \leq 250)$, which is the number of lanes on the highway. The lanes are numbered $1,2, \ldots, m$.

The second line of input contains two integers $k\left(1 \leq k \leq 10^{6}\right)$ and $r\left(1 \leq r \leq 10^{6}\right)$, which are the lane changing parameters.

The next $n$ lines describe the straightaways in order. Each of these lines contains a single integer $\ell(1 \leq \ell \leq$ $10^{6}$ ), which is the length of this straightaway.

The next $n-1$ lines describe the curves in order. Each of these lines contains two integers $s\left(1 \leq s \leq 10^{6}\right)$, which is the stretch of this curve, and $c\left(-10^{6} \leq c \leq 10^{6}\right)$, which is the curvature of this curve. It is guaranteed that $s+c \cdot m>0$.

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The $i$ th curve connects the $i$ th and $(i+1)$ th straightaway.

## Output

Display the minimum distance Sam must travel.

Sample Input 1

## Sample Output 1

| 4 | 3 | 51 |
| :--- | :--- | :--- |
| 5 | 2 |  |
| 10 |  |  |
| 10 |  |  |
| 10 |  |  |
| 10 |  |  |
| 4 | -1 |  |
| 4 | -1 |  |
| 4 | 1 |  |

Sample Input 2
Sample Output 2

| 4 | 3 | 6 |
| :--- | :--- | :--- |
| 5 | 2 |  |
| 10 |  |  |
| 10 |  |  |
| 10 |  |  |
| 10 |  |  |
| 10 | -3 |  |
| 10 | -3 |  |
| 10 | 1 |  |

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# Problem F Treasure Spotting <br> Time limit: 1 second 

For Timmy's birthday his parents threw him a pirate themed party! A treasure is buried in the yard and now it is up to Timmy and his pirate crew to find it. Help the pirates find the treasure by letting them know who can see where the treasure is buried.

To make the game interesting, there are walls placed in the yard to obscure vision. Each pirate has a field of view that determines what they can see. Each pirate can see a certain distance away and can only see in a semi-circle based on the direction they are looking (see image below). A point cannot be seen in a pirate's field of view if either another pirate or some part of a wall is directly between the point that is being looked at and the pirate that is looking. Each pirate is a single point, and each wall is an infinitely thin line.

Which pirates can see where the treasure is buried?


Figure F.1: The left picture illustrates Sample Input 1 where the right-most pirate is the only one who can see the location of the buried treasure. The right picture illustrates Sample Input 2 where the middle pirate is the only one who can see the buried treasure.

## Input

The first line of input contains two integers $W$ ( $0 \leq W \leq 1000$ ), which is the number of walls and $P(1 \leq P \leq 1000)$, which is the number of pirates.

The second line contains the coordinates of the treasure.
The next $W$ lines describe the walls. Each of these lines contains two coordinates $(x, y)$ and $\left(x^{\prime}, y^{\prime}\right)$ which are the two (distinct) endpoints of this wall.

The next $P$ lines describe the pirates. The $i$ th of these lines contains two (distinct) coordinates $\left(x_{i}, y_{i}\right)$, which is the position of the $i$ th pirate, and $\left(x_{i}^{\prime}, y_{i}^{\prime}\right)$, which is the furthest point that this pirate can see in the direction they are looking. That is, the radius of the semi-circle for this pirate is the distance between $\left(x_{i}, y_{i}\right)$ and $\left(x_{i}^{\prime}, y_{i}^{\prime}\right)$.

All coordinates are an $(x, y)$ integer pair with $|x|,|y| \leq 10^{9}$. No two pirates will have the same coordinate position, the treasure will not share a coordinate position with any pirate and no part of any wall will touch a pirate or the treasure. Note that walls can overlap in any way with other walls.

## Output

Display $P$ lines, one per pirate. The $i$ th of these lines should display visible if the $i$ th pirate can see where the treasure is buried and not visible otherwise.

| Sample Input 1 | Sample Output 1 |
| :---: | :---: |
| 23 | not visible <br> visible <br> not visible |
| 23 |  |
| 1220 |  |
| 0031 |  |
| $\begin{array}{lllll}0 & 1 & 3\end{array}$ |  |
| 5055 |  |
| 2625 |  |

Sample Input 2

## Sample Output 2

```
O
0
1 0
3
-2 0
```


# Problem G Neighborhood Watch <br> Time limit: 1 second 

Jennifer was nominated to be neighborhood watch captain and is now in charge of managing the watch for her street.

Jennifer's street consists of houses on only one side of the road. She has a plan of which houses will be a neighborhood watch house and wants to know how safe the plan is. A walk from one house to another house (not necessarily distinct) is considered safe if there is at least one house along the walk that is a neighborhood watch house. The safety rating of a plan is the number of walks that are safe on the street. Since a walk is either safe or not safe, when traveling in either direction, it is not counted twice in the safety rating.


Figure G.1: Sample input. One example of a safe walk is traveling from house 1 to house 5 .

Tell Jennifer the safety rating of her plan.

## Input

The first line of input contains two integers $N(1 \leq N \leq 200000)$, which is the number of houses on the street, and $K(0 \leq K \leq N)$, which is the number of neighborhood watch houses in Jennifer's plan. The houses are numbered $1, \ldots, N$.

The next $K$ lines describe the neighborhood watch houses. Each of these lines contains a single integer $H(1 \leq H \leq N)$, which is the house number of a neighborhood watch house. The house numbers are given in strictly increasing order.

## Output

Display the safety rating of Jennifer's plan.


# Problem H Small Schedule <br> Time limit: 1 second 

Everybody is into cloud computing these days, so quite a few different business models are being experimented with. You are trying a very simple one: you sell time on your machines in one of two batches called slots. A customer can buy one second of CPU time or $Q$ seconds for some integer $Q$.

Each time slot a customer purchases must be completed on a single machine, but you get to decide how to allocate the purchased time
 slots between machines.

After coming back from a long vacation, you see that all of your machines are idle and a variety of orders have come in. To keep customers happy, you must decide how to distribute these requests between machines in a way that minimizes the time when the purchased time slots are finally all completed.

What is the smallest amount of time in which you can complete all of the purchased time slots?

## Input

The input consists of a single line containing four integers $Q(2 \leq Q \leq 1000)$, which is the time needed to complete the longer batches, $M(1 \leq M \leq 1000000)$, which is the number of machines owned by your company, $S(0 \leq S \leq 1000000)$, which is the number of 1-second time slots purchased, and $L(0 \leq L \leq$ 1000000 ), which is the number of $Q$-second time slots purchased.

## Output

Display the smallest amount of time in which you can complete all of the purchased time slots.

| Sample Input 1 | Sample Output 1 |
| :---: | :---: |
| 2436 | 4 |
| Sample Input 2 | Sample Output 2 |
| 3435 | 6 |
| Sample Input 3 | Sample Output 3 |
| 10201 | 10 |

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# Problem I <br> Mr. Plow King <br> Time limit: 2 seconds 

In the kingdom of Winterfield, there are several cities and each pair of cities is connected by exactly one old dirt road. The king of Winterfield has decided that he will upgrade some of those roads. The set of roads that he upgrades must be such that it is possible to get from any city in the kingdom to any other city via some sequence of upgraded roads.

Because there is so much snow in Winterfield, the king has also decided to plow some of those upgraded roads. The local plow company, Mr. Plow, and the king have agreed on the following: the king labels each of the upgraded roads $1,2, \ldots, m$ (the label of each road is the number of gold pieces it costs to plow that road) and each road must receive a distinct label. Mr. Plow will plow a set of upgraded roads so that it is possible to get from any city to any other city via some sequence of plowed roads. Mr. Plow will select the cheapest possible set of roads that satisfies the above condition.

For example, if the kingdom has six cities and the king decides to upgrade and label the 8 bolded dirt roads as follows, then Mr. Plow will then plow the roads with labels 1, 2, 3, 4 and 6 (costing a total of 16 gold pieces).


The king has decided on the number of roads to upgrade, but he is not sure how to label them, so he has turned to Barney (the kingdom's mathematician) to help decide. However, the king is unaware that Barney is actually invested in Mr. Plow, so Barney will choose the set of roads to upgrade and how to label them so that the total cost is as large as possible. What is the maximum cost of plowing the roads?

## Input

The input consists of a single line containing two integers $n$ ( $2 \leq n \leq 1000000$ ), which is the number of cities, and $m\left(n-1 \leq m \leq \frac{n(n-1)}{2}\right)$, which is the number of roads to be upgraded.

Output
Display the maximum possible cost of plowing the roads following the rules above.

| Sample Input 1 | Sample Output 1 |  |  |
| :--- | :--- | :---: | :---: |
| 43 | 6 |  |  |
| Sample Input 2 |  |  | Sample Output 2 |
| 68 | 22 |  |  |
|  |  |  |  |
| Sample Input 3 | Sample Output 3 |  |  |
| 912 | 56 |  |  |

## Problem J Rainbow Road Race <br> Time limit: 2 seconds

Marcy is attending Pride Fest and has signed up to participate in the Rainbow Road Race. On each street, there are volunteers who have colored chalk powder. As contestants walk along a street, volunteers throw chalk onto the contestants. The chalk thrown on each street is one of the seven colors of the rainbow (Red, Orange, Yellow, Green, Blue, Indigo, Violet). Once a person starts walking along a street, they must walk to the end of that street.

The race starts at the Festival Tent. The goal of the race is to get covered by chalk of every color and come back to the tent. Help Marcy determine the shortest distance she has to travel to get every color and make it back to the tent.


Figure J.1: The left picture illustrates Sample Input 1 and the right picture illustrates Sample Input 2. The triangle is the Festival Tent.

## Input

The first line of input contains two integers $n\left(7 \leq n \leq 7^{3}\right)$, which is the number of fun locations at the festival, and $m\left(7 \leq m \leq 7^{4}\right)$, which is the number of streets connecting the fun locations. The fun locations are numbered $1, \ldots, n$ and the Festival Tent is location 1.

The next $m$ lines describe the streets. Each of these lines contains three integers $\ell_{1}, \ell_{2}\left(1 \leq \ell_{1}<\ell_{2} \leq n\right)$ and $d\left(1 \leq d \leq 7^{5}\right)$, followed by a single character $c(c$ is one of $\mathrm{R}, \mathrm{O}, \mathrm{Y}, \mathrm{G}, \mathrm{B}, \mathrm{I}, \mathrm{V})$. This specifies that this street connects locations $\ell_{1}$ and $\ell_{2}$, is $d$ meters long and the chalk thrown is color $c$.

It is always possible to travel between any pair of fun locations. There is at most one street between any two pairs of locations and each color appears at least once.

## Output

Display the shortest distance Marcy has to travel to get every color and make it back to the Festival Tent.

| Sample Input 1 | Sample Output 1 |
| :--- | :--- | :--- | :--- |
| 7 7  7 <br> 1 2 1 R <br> 2 3 1 0 <br> 3 4 1 $Y$ <br> 4 5 1 G <br> 5 6 1 B <br> 6 7 1 I <br> 1 7 1 V |  |

## Sample Input 2

## Sample Output 2

| 8 | 7 |  | 14 |
| :--- | :--- | :--- | :--- |
| 1 | 2 | 1 | $R$ |
| 1 | 3 | 1 | 0 |
| 1 | 4 | 1 | $Y$ |
| 1 | 5 | G |  |
| 1 | 6 | 1 | $B$ |
| 1 | 7 | 1 | I |
| 1 | 8 | 1 | V |

